

2.1 Derivative Basics: Form B Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_set.py`). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

General grading rules

- **Answer-only items** (Q1, Q9, Q13): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse) earns 0 for that part.
- **Show-work items** (Parts II–IV other than Q9): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form B

Item	Answer	Rubric / trap
1(a)	3	<i>Trap:</i> the tangent drawn there invites the slope, 2; earns 0.
1(b)	-2	<i>Trap:</i> giving the height $f(-3) = 3$; earns 0.
1(c)	0	<i>Trap:</i> giving the height 1 of the low point; earns 0.
1(d)	DNE	Corner at $(-1, 3)$: one-sided slopes 2 and -2 . <i>Trap:</i> 0, because $(-1, 3)$ is a high point.
1(e)	$(-2, -1)$ and $(1, 6)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-2, 6)$, which includes the corner and the falling stretch $(-1, 1)$.
1(f)	$f'(-3) < 0 < f'(3)$	$f'(-3) = -2$, $f'(3) = 2$. 3 pts, all or nothing.
2	$y = -\frac{1}{2}(x - 4)$, i.e. $y = -\frac{1}{2}x + 2$	1 pt $f(4) = 0$; 2 pts $f'(x)$; 1 pt $f'(4) = -\frac{1}{2}$; 1 pt equation in any correct form. <i>Trap:</i> $f'(x) = x^{-1/2} - 1$, so $f'(4) = \frac{1}{2} - 1 = -\frac{1}{2}$. The point $(4, 0)$ is on the x -axis, so the equation starts $y - 0$, which students distrust. Writing $f'(4) = \frac{1}{2}$ (forgetting the -1) earns 2 of 5.
3	$\frac{3}{5}$	2 pts $q(3) = 5$; 3 pts slope $\frac{5-2}{3-(-2)} = \frac{3}{5}$. <i>Trap:</i> q needs the chain rule, which has not been taught, so the given point is the only route. Students who try to differentiate q stall. Sign slips with $3 - (-2)$ earn 2.
4(a)	4	3 pts. Simplifies to $y = 4x + 1$. <i>Trap:</i> 6 (the coefficient before dividing by 2) or 3; earns 0.
4(b)	-3	2 pts. Substitute $x = -1$. <i>Trap:</i> 0 or -2 , from reading pieces of the equation; earns 0.
5	-4	<i>Trap:</i> g' is printed and invites $g'(-1) = 4$; earns 0.
6	$f'(x) = -\frac{5}{(x+1)^2}$	2 pts $f(x+h) = \frac{5}{x+h+1}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $5(x+1) - 5(x+h+1) = -5h$; 2 pts cancel h to get $\frac{-5}{(x+h+1)(x+1)}$; 2 pts let $h \rightarrow 0$. <i>Trap:</i> writing $f(x)+h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	1	1 pt $g(-1) = 1$; 1 pt setup; 3 pts factoring out $x+1$ to get x^2-x-1 , by any valid method. Synthetic or long division by $x+1$ works (on 1, 0, -2 , -1 , with the zero placeholder). So does grouping with the sum of cubes: $x^3 - 2x - 1 = (x^3 + 1) - 2(x + 1) = (x + 1)(x^2 - x + 1 - 2) = (x + 1)(x^2 - x - 1)$. No particular method is required. 3 pts value. <i>Trap</i> : the denominator is $x - (-1) = x + 1$; writing $x - 1$ gives a limit that does not exist. A student who divides and drops the $0x^2$ placeholder gets a wrong quotient. Rules only: at most 2.
8(a)	$-\frac{1}{6}$	$-f'(9)$ for $f(x) = \sqrt{x}$: $-\frac{1}{2\sqrt{9}} = -\frac{1}{6}$. <i>Trap</i> : $\frac{1}{6}$, ignoring the $-h$; earns 0. 3 pts for recognizing which derivative the limit is (or a correct algebraic setup), 3 pts for the value.
8(b)	0	$\cos x + 1 = \cos x - \cos \pi$, so this is the derivative of $\cos$ at π : $-\sin \pi = 0$. <i>Trap</i> : “0/0, DNE” or 1; earns 0. 3 pts for recognizing which derivative the limit is (or a correct algebraic setup), 3 pts for the value.
9	(a) 0 (b) 0	3 each. <i>Traps</i> : (a) $-\sin(\pi/3)$; (b) $g'(-1) = 5$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says “for every x ”.
10(a)	$-16x^{-3} + 3 \cos x$	1 pt rewriting $8x^{-2}$; 2 pts derivative. <i>Trap</i> : $-2 \cdot 8 = -16$; the exponent -3 .
10(b)	1	1 pt rewriting $x - 4$ (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : differentiating top and bottom separately gives $\frac{3x^2-8x}{2x}$; earns 0.
10(c)	$2x^{-1/2} - \frac{3}{2}x^{1/2}$	1 pt rewriting $4x^{1/2} - x^{3/2}$; 2 pts derivative. <i>Trap</i> : $x\sqrt{x} = x^{3/2}$, not $x^{1/2}$.
10(d)	$2x - 3 + 2 \sin x$	1 pt rewriting $x^2 - 3x - 10 - 2 \cos x$; 2 pts derivative. <i>Trap</i> : the derivative of $-2 \cos x$ is $+2 \sin x$.
11	$\frac{3\pi^2}{4} - 4$	3 pts $f'(x) = -4 \sin x + 3x^2$; 2 pts value. <i>Trap</i> : $\sin \frac{\pi}{2} = 1$; $3 \left(\frac{\pi}{2}\right)^2 = \frac{3\pi^2}{4}$. Writing $\frac{3\pi}{2}$ for the second term earns 3.
12	$x = \frac{\pi}{3}$	2 pts $f'(x) = 1 - 2 \cos x$; 2 pts $\cos x = \frac{1}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : $\frac{5\pi}{3}$ also solves $\cos x = \frac{1}{2}$ but lies outside $[0, \pi)$; listing it costs 2.
13(a)	IV	u' is a line crossing zero at $x = \frac{1}{2}$, where u turns, with the sign of u 's slope on each side. <i>Trap</i> : III, the same line with the opposite sign, or a parabola because u is a parabola.
13(b)	II	v' is a parabola with zeros at $x = -\frac{5}{2}$ and $x = \frac{1}{2}$, where v turns. <i>Trap</i> : I, which has the right zeros and the wrong signs.
14	see sketch description	$k' = 0$ for $x < -2$; $k' = -1$ for $-2 < x < 1$; $k' = 2$ for $x > 1$. Open circles at the ends of each segment, at $x = -2$ and $x = 1$. 2 pts per horizontal piece (correct height and interval); -1 once if the open circles are missing or replaced by closed dots. <i>Trap</i> : drawing k 's shape again, or joining the pieces with vertical segments.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 12 + 6 + 12 + 5 + 6 + 6 + 6 = 100$.